

PART I: AN INTRODUCTION TO LEAST-SQUARES MONTE CARLO SIMULATION

During the past few years there has been an increased demand for sophisticated risk modelling. This has been driven both by new and more demanding regulatory frameworks, as well as opportunities for enhancing profitability through better understanding of what drives the various risks facing a business. Regardless of underlying reason, the methods and supporting technology become increasingly complex as they need to fulfil requirements of accuracy and precision (O'Donnell, 2010).

BACKGROUND

Following new regulatory requirements, modelling financial risk grows in complexity as it faces tighter constraints and higher uncertainties. As existing models and methods reach their limitations, we turn to simulation-based models. That is, models that aim to simulate possible scenarios for the evolution of certain risk factors. By generating a sufficiently large number of potential scenarios one can estimate the distribution of future profits and losses over the time horizon that the scenarios are generated. From the estimated distribution it is then possible to calculate an expected shortfall or another suitable measure of risk, e.g. Value-at-Risk.

A NEED FOR SIMULATION-BASED MODELLING

Insurance policies, complex derivatives and bonds with embedded optionality are examples of financial products that virtually always require numerical procedures in order to determine reliable theoretical valuations. Monte Carlo (MC) simulation is a standard approach when valuing these types of products. However, using a straight forward MC approach is not applicable in certain settings. Due to the fact that these types of financial products are complex and path-dependent, the challenge of valuation leads to *nested simulations*, where a number of economic scenarios each branch out to form a number of valuation scenarios.

NESTED SIMULATION

Nested simulations are performed in a two-step procedure. First, scenarios are generated to obtain distributions of all the risk factors affecting pricing up to a suitable time horizon. Secondly, a re-pricing is made (based on the previously obtained information) at the end of the time horizon for each previously generated scenario (Gordy & Juneja, 2008). The scenarios

gained in the first step are referred to as *outer* scenarios and the second step generations are called *inner* scenarios. A nested simulation is exhibited on the left-hand side in Figure 1 below.

To give an example of a nested simulation, consider the calculation of capital requirement under Solvency II. Insurers typically use between 1,000 and 10,000 outer scenarios to value their balance sheets. With 10,000 outer scenarios, the total number of scenarios is likely to be at least 10 million for a nested calculation (Morrison, 2009). Therefore, the technique is characterised by time-consuming computations and we start to question if nested simulations are practically applicable, despite high-quality end results. One way of addressing the long computation time is to use an extension of the basic Monte Carlo method called Least-Squares Monte Carlo (LSMC) (Longstaff & Schwartz, 2001).

THE BASICS OF LEAST-SQUARES MONTE CARLO

Fundamentally, LSMC is a simulation method that combines a Monte Carlo simulation with a least-squares regression. The aim of the regression varies with the application. In general the goal is to take advantage of information obtained when generating the outer scenarios in order to reduce the total number of scenarios, but nevertheless retain an accurate valuation.

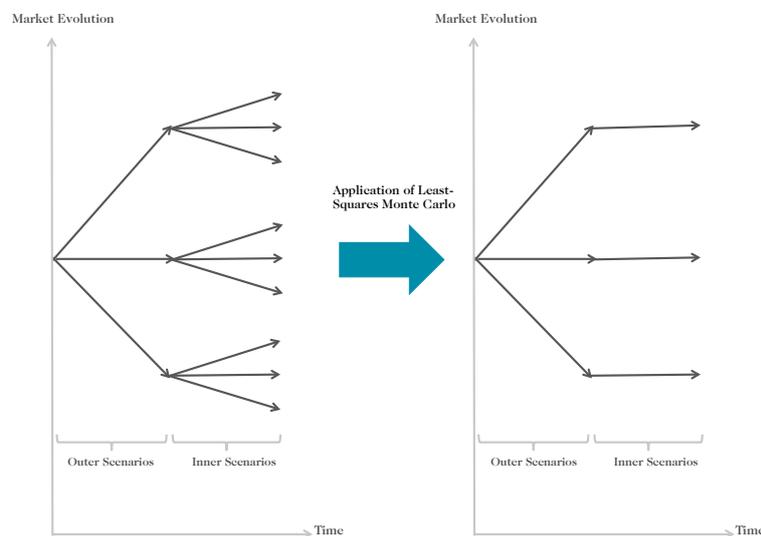


Figure 1: Simplification of a nested model by applying LSMC simulation

SIMPLIFICATION WHEN USING LEAST-SQUARES MONTE CARLO

When LSMC is applied to situations that require simulation-based techniques, the regression is used to estimate parameters of a proxy function that calculates the future values that the inner scenarios normally would give. In this way, LSMC eliminates the need for a large fraction of the inner simulations and therefore reduces the calculation time.

To further explain the point of LSMC, consider a nested simulation as illustrated on the left-hand side in Figure 1 on the previous page. In the figure we can see that each of the outer scenarios is used as a starting point for a set of inner scenarios. In a nested simulation the subject for valuation, e.g. a portfolio, is valued as the discounted mean of all the simulated scenarios. As stated above, when using a LSMC approach the number of inner scenarios is highly reduced, resulting in a simulation as depicted on the right-hand side in Figure 1 (Morrison, 2009).

REGRESSION IN LEAST-SQUARES MONTE CARLO

To compensate for the inaccuracy that occurs when we restrict our model to being based on a small number of inner scenarios, a regression¹ is made over all the inaccurate valuations. However, instead of using the inaccurate values for estimation we use a fitted regression function, as shown in Figure 2. By using a fitted function, we can take advantage of the spread between errors that will appear due to outer scenario generation to minimise inaccuracy. The fitted function can then be used for valuation by assessing the function for the different outer scenarios.

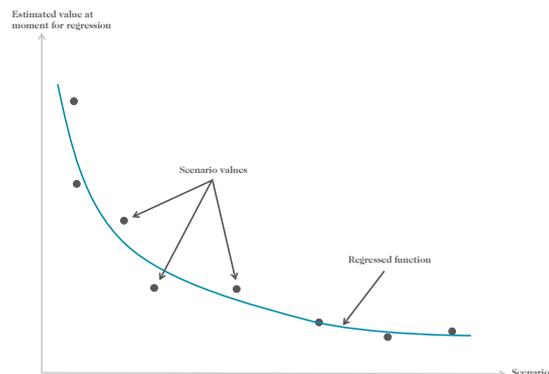


Figure 2: Fitted regression function used in LSMC simulation

¹The regression function used is made up of terms representing the underlying risk factors in the valuation, such as underlying asset returns, volatilities or interest rates.

SUMMARY

In this article we have briefly explained a recognised technique for sophisticated risk modelling, LSMC. This simulation-based method is used to simplify valuations that require nested simulations to be reliable and accurate. A key aspect of LSMC is the regression function used to reduce the number of inner scenarios, as its composition can affect the performance of the simulation technique. The construction of regression functions and means of solving and optimising the regression to further improve performance will be explained in "Part II: Regression Functions in Least-Squares Monte Carlo Simulations".

References

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CONTACT US

Do you want to know more about Kidbrooke Advisory?
Please contact us for more information.

e. info@kidbrooke-advisory.com

a. Kidbrooke Advisory Aktiebolag
Birger Jarlsåatan 20
SE-114 34 Stockholm
Sweden



FREDRIK DAVÉUS
Managing Partner
+46(0)761 170 009
fredrik.daveus@kidbrooke.com



EDVARD SJÖGREN
Partner
+46(0)733 182 583
edvard.sjogren@kidbrooke.com

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